LINEAR CIRCUIT ANALYSIS (EED) – U.E.T. TAXILA

06

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INTRODUCTION

Capacitor and inductor are two important passive elements.

Capacitors and inductors don’t dissipate energy but store energy which can be retrieved later.

That’s why the capacitors and inductors are called energy storage elements.

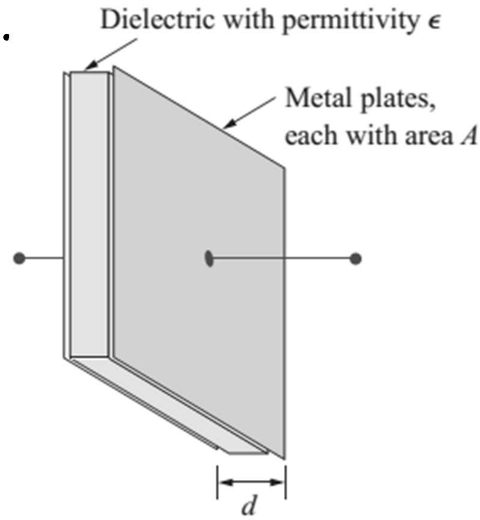
Capacitors and inductors can also be connected in series and parallel in electrical circuits.

CAPACITOR

A capacitor is a passive element designed to store energy in its electric field.

Capacitors are used extensively in electronics, communications and power systems.

A Capacitor consists of two conducting plates separated by an insulator (dielectric).



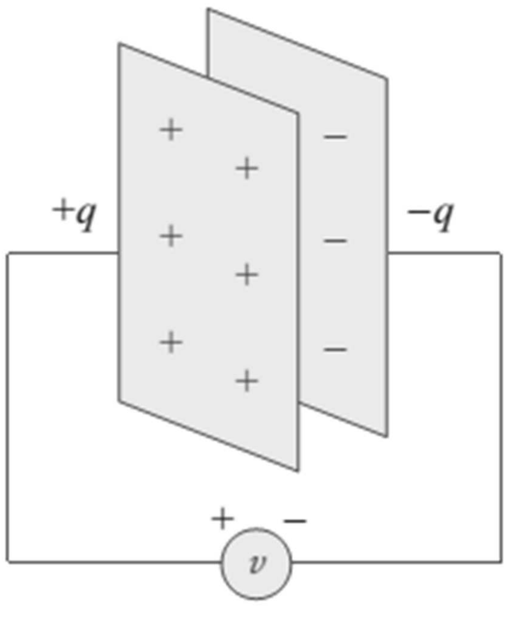
CAPACITOR

In practical, plates may be aluminium foil while the dielectric may be air, ceramic, paper or mica.

When a voltage source is connected to the capacitor, the source deposits a positive charge on one plate and negative charge on the other.

The charge stored in capacitor is directly proportional to the applied voltage.

$$q = Cv$$



CAPACITOR

Where ‘C’ is constant of proportionality, known as capacitance of the capacitor.

Capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in Farad (F).

$$1 \text{ farad} = 1 \text{ coulomb/volt}.$$

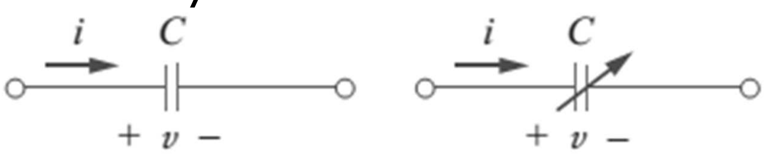
The capacitance of the capacitor is directly proportional to the area of plates and inversely proportional to the distance between the plates.

$$C = \frac{\epsilon A}{d}$$

CAPACITOR

Where ‘ϵ’ is constant of proportionality, known as permittivity of dielectric material.

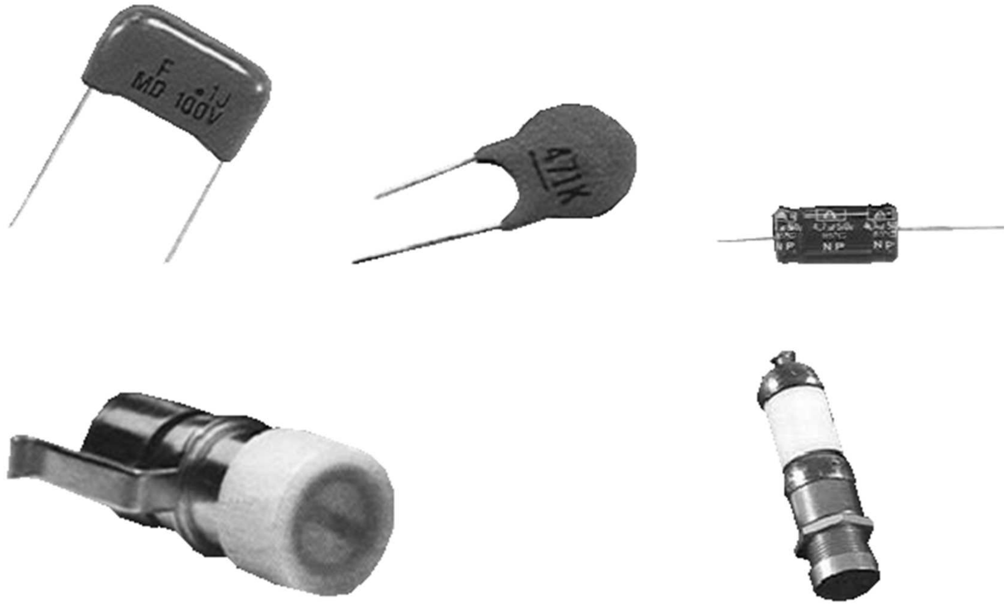
The capacitors may be of fixed value or variable value type.



The passive sign convention is equally applicable for capacitors.

According to Passive Sign Convention: if $v>0$ & $i>0$ or $v<0$ & $i<0$, capacitor is being charged, and if v or $i<0$, the capacitor is discharging.

CAPACITOR

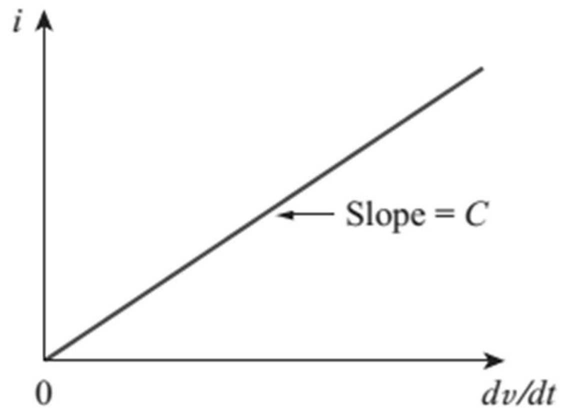


CAPACITOR

Differentiating both sides of capacitor equation;

$$i = C\frac{dv}{dt}$$

This is current-voltage relationship for a capacitor.



CAPACITOR

The capacitor obeying current-voltage relationship straight line is known as Linear Capacitor.

Capacitor voltage can also be calculated;

$$v = \frac{1}{C}\int_{-\infty}^{t} i \, dt$$

$$v = \frac{1}{C}\int_{t_0}^{t} i \, dt + v(t_0)$$

where $v(t_0) = q(t_0)/C$

CAPACITOR

The instantaneous power delivered to capacitor is;

$$p = vi = Cv\frac{dv}{dt}$$

The energy stored in capacitor is;

$$w = \int_{-\infty}^{t} p \, dt$$

$$w = \frac{1}{2}Cv^2 \qquad w = \frac{q^2}{2C}$$

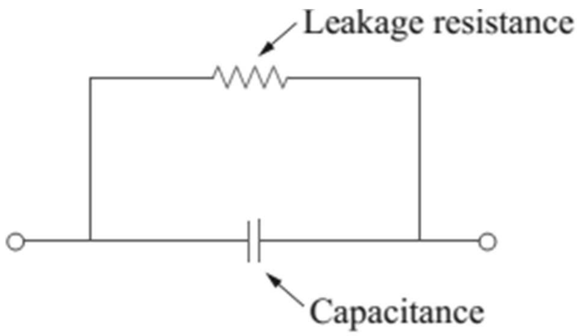
CAPACITOR

A capacitor is an open circuit to dc supply.

$$i = C\frac{dv}{dt}$$

The voltage on a capacitor cannot change abruptly.

A real non-ideal capacitor is modelled as shown.



PROBLEMS

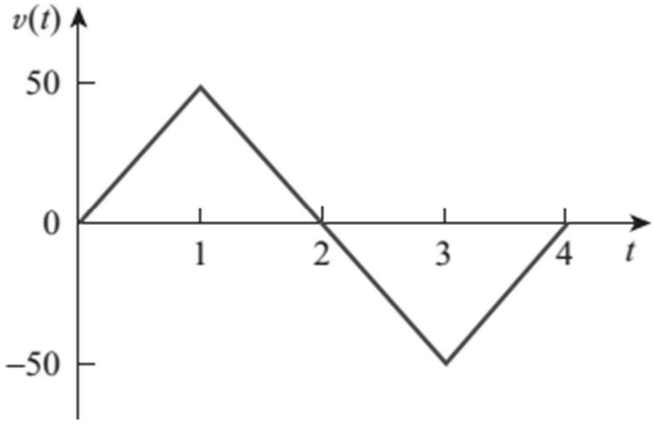
Calculate the charge stored on a 3 pF capacitor with 20 V across it? Also find energy? (60 pC, 600 pJ)

The voltage across a 5 pF capacitor is $v(t)=10\cos 6000t$, calculate the current? (-0.3sin6000t A)

Determine the voltage across a 2 μF capacitor with current $i(t)=6e^{-3000t}$ mA? Assume that initial capacitor voltage is zero. ($1-e^{-3000t}$ V)

PROBLEMS

Determine the current through a 200 μF capacitor whose voltage waveform is shown;

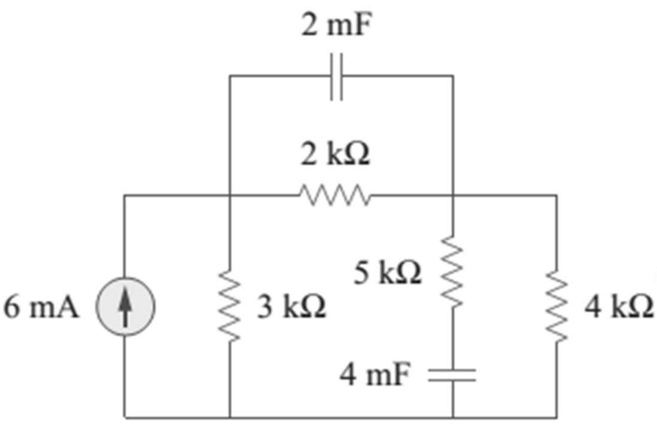


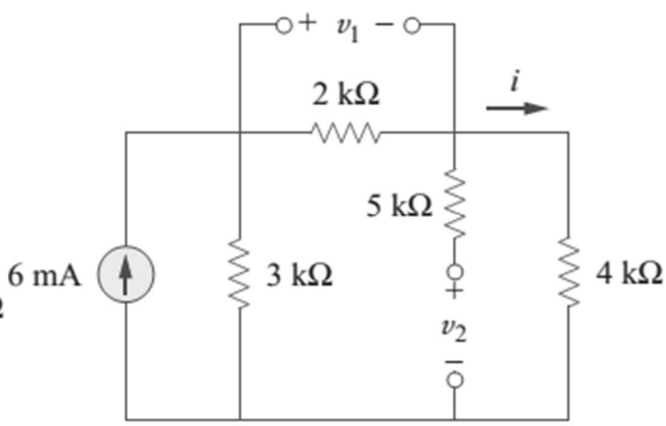
$$v(t) = \begin{cases} 50t \text{ V} & 0 < t < 1 \\ 100 - 50t \text{ V} & 1 < t < 3 \\ -200 + 50t \text{ V} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

(10 mA, -10 mA, 10 mA, 0)

PROBLEMS

Obtain the energy stored in each capacitor?

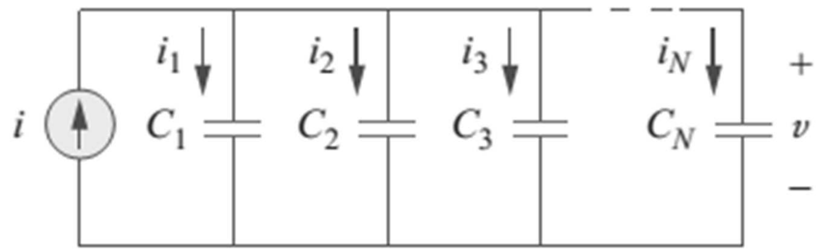




(16 mJ, 128 mJ)

SERIES AND PARALLEL CAPACITORS

First consider parallel capacitors;



Applying KCL; $i = i_1 + i_2 + i_3 + \cdots + i_N$

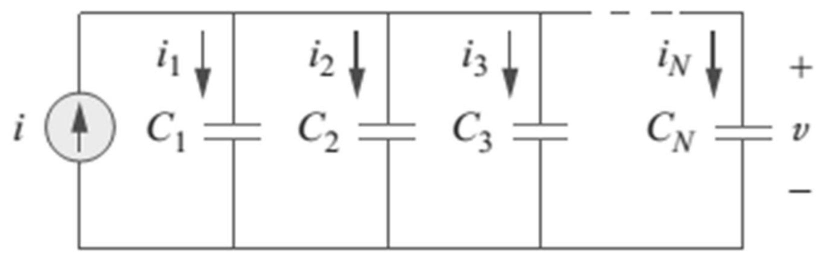
$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + C_3\frac{dv}{dt} + \cdots + C_N\frac{dv}{dt}$$

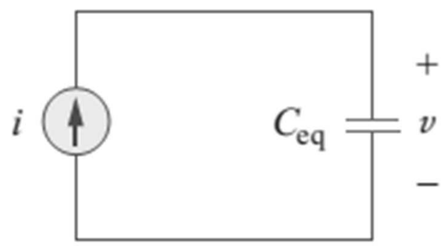
$$= \left(\sum_{k=1}^{N} C_k\right)\frac{dv}{dt} = C_{eq}\frac{dv}{dt}$$

$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N$$

SERIES AND PARALLEL CAPACITORS

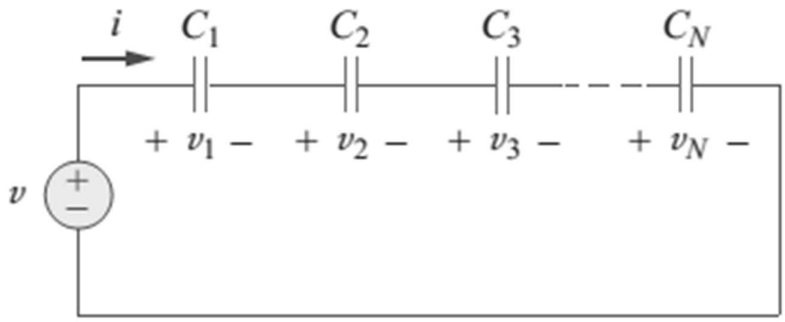
The equivalent capacitance of N Parallel connected capacitors is the sum of the individual capacitances.





SERIES AND PARALLEL CAPACITORS

Consider series capacitors;



Applying KVL;

$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

Putting capacitor voltages;

SERIES AND PARALLEL CAPACITORS

$$v = \frac{1}{C_1}\int_{t_0}^{t} i(t)\,dt + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i(t)\,dt + v_2(t_0)$$

$$+ \cdots + \frac{1}{C_N}\int_{t_0}^{t} i(t)\,dt + v_N(t_0)$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}\right)\int_{t_0}^{t} i(t)\,dt + v_1(t_0) + v_2(t_0)$$

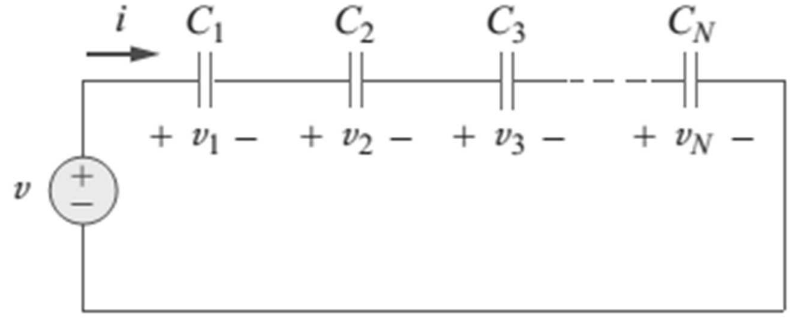
$$+ \cdots + v_N(t_0)$$

$$= \frac{1}{C_{eq}}\int_{t_0}^{t} i(t)\,dt + v(t_0)$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}$$

SERIES AND PARALLEL CAPACITORS

The equivalent capacitance of Series connected capacitors is the reciprocal of the sums of the reciprocals of the individual capacitances.

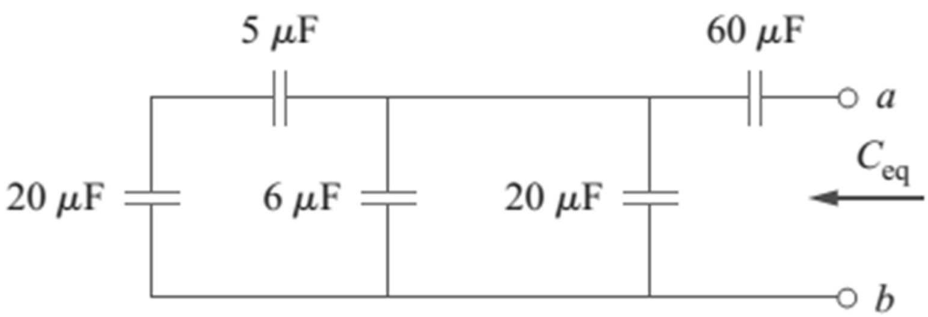


For two capacitors connected in series;

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

PROBLEMS

Find equivalent capacitance?



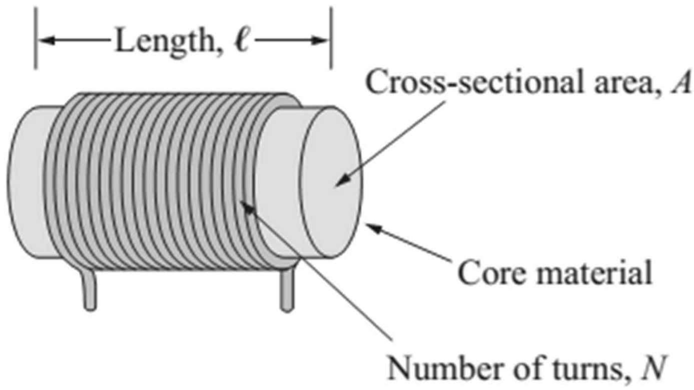
(20 μF)

INDUCTOR

An inductor is a passive element designed to store energy in its magnetic field.

Inductors are used extensively in power systems, electrical machines and electronics.

An Inductor consists of a coil of conducting wire.



INDUCTOR

The voltage across the inductor is directly proportional to the time rate of change of current through it.

$$v = L\frac{di}{dt}$$

Where 'L' is constant of proportionality, known as inductance of inductor.

Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in Henry (H).

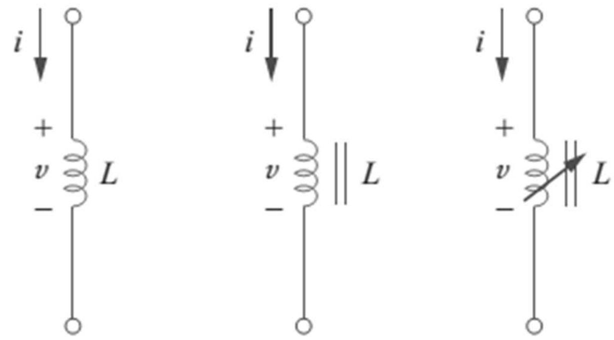
INDUCTOR

The inductance of inductor is directly proportional to the number of turns & area and inversely proportional to the length.

$$L = \frac{N^2 \mu A}{\ell}$$

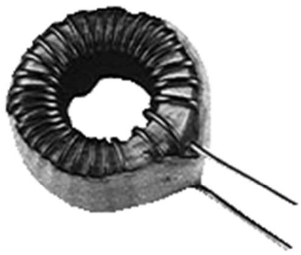
Where 'µ' is the constant of proportionality, known as permeability of core material.

Inductors may be of fixed value or variable value.



INDUCTOR

(a)



(b)

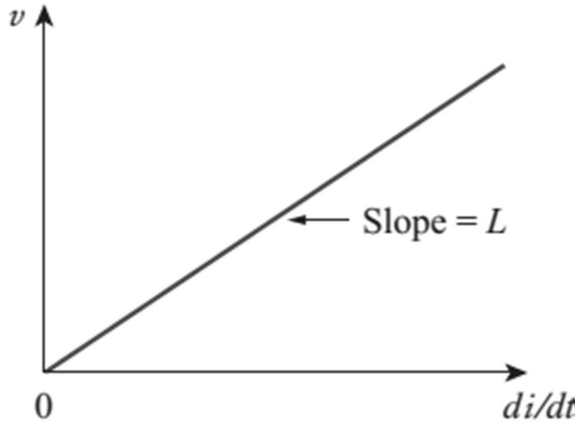
(c)

INDUCTOR

The current-voltage relationship for inductor;

$$v = L\frac{di}{dt}$$

Inductor obeying straight line in current-voltage is known as linear inductor.



INDUCTOR

Integrating inductor equation gives current;

$$i = \frac{1}{L}\int_{-\infty}^{t} v(t)\,dt$$

$$i = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$$

The power delivered to inductor is;

$$p = vi = \left(L\frac{di}{dt}\right)i$$

INDUCTOR

The energy stored in inductor is;

$$w = \int_{-\infty}^{t} p\,dt = \int_{-\infty}^{t} \left(L\frac{di}{dt}\right) i\,dt$$

$$= L\int_{-\infty}^{t} i\,di = \frac{1}{2}Li^2(t) - \frac{1}{2}Li^2(-\infty)$$

$$w = \frac{1}{2}Li^2$$

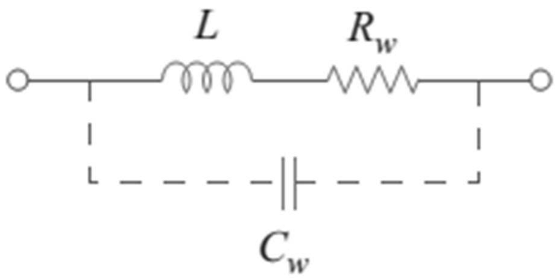
An inductor acts like a short circuit to dc supply.

$$v = L\frac{di}{dt}$$

INDUCTOR

The current though an inductor cannot change instantaneously.

A practical, non-ideal inductor can be modelled as shown.



PROBLEMS

The current through a 0.1 H inductor is $i(t)=10te^{-5t}$ A. Find voltage across inductor and energy stored?

[$e^{-5t}(1-5t)$ V, $5t^2e^{-10t}$ J]

Find the current through a 5 H inductor, if the voltage is;

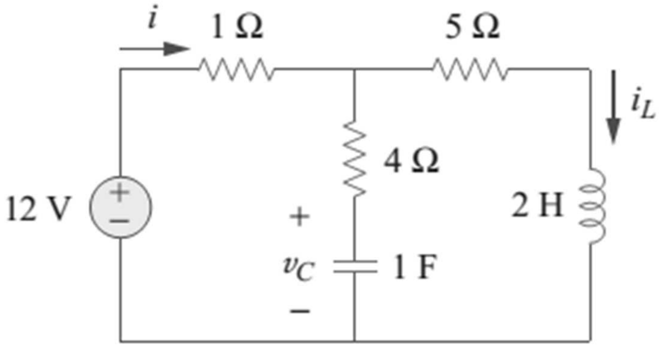
$$v(t) = \begin{cases} 30t^2, & t > 0 \\ 0, & t < 0 \end{cases}$$

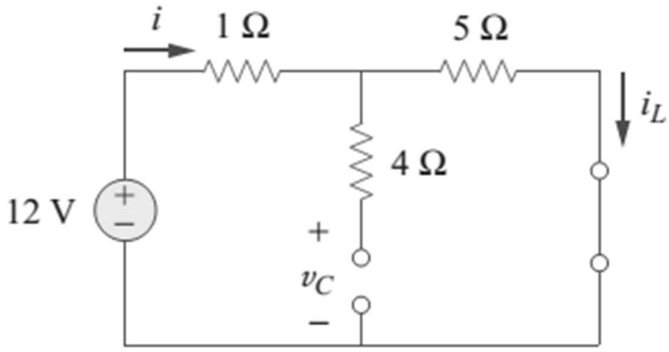
Also find energy at t=5 s?

($2t^3$ A, 156.25 kJ)

PROBLEMS

Under dc conditions, find i, v_c and i_L? Also find energy?

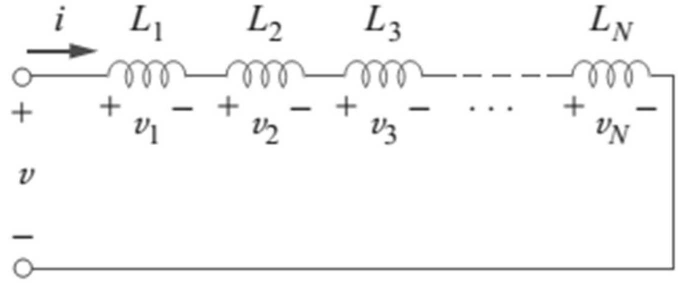




(2 A, 2 A, 10 V, 50 J, 4 J)

SERIES AND PARALLEL INDUCTORS

First consider inductors are connected in series;



Applying KVL; $v = v_1 + v_2 + v_3 + \cdots + v_N$

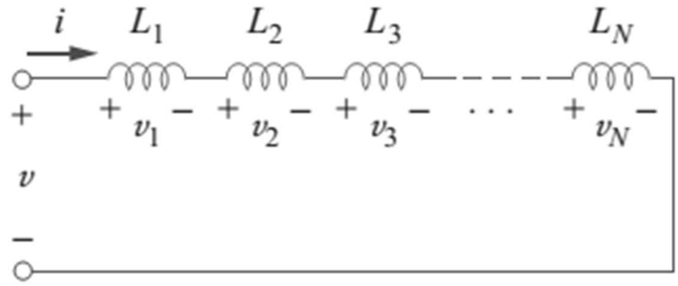
$$v = L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt} + \cdots + L_N\frac{di}{dt}$$

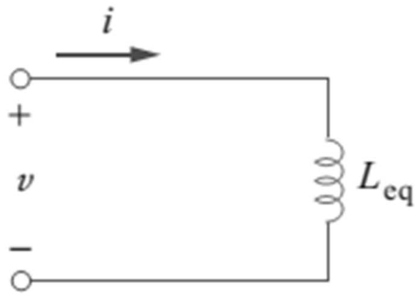
$$= \left(\sum_{k=1}^{N} L_k\right)\frac{di}{dt} = L_{eq}\frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N$$

SERIES AND PARALLEL INDUCTORS

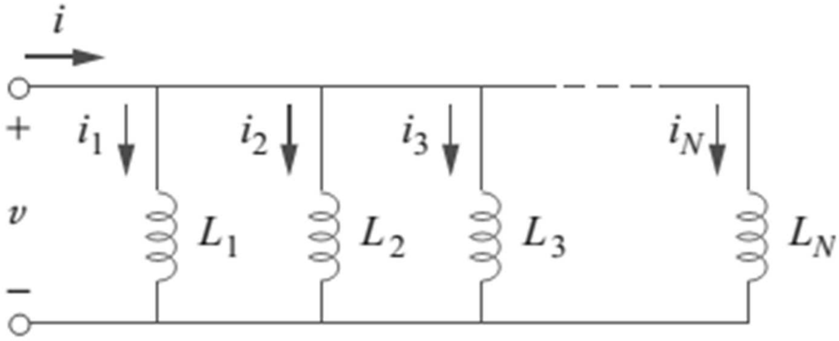
The equivalent inductance of Series connected inductors is the sum of the individual inductances.





SERIES AND PARALLEL INDUCTORS

Consider inductors are connected in parallel;



Applying KCL;

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

Putting inductors currents;

SERIES AND PARALLEL INDUCTORS

$$i = \frac{1}{L_1}\int_{t_0}^{t} v\,dt + i_1(t_0) + \frac{1}{L_2}\int_{t_0}^{t} v\,dt + i_2(t_0)$$

$$+ \cdots + \frac{1}{L_N}\int_{t_0}^{t} v\,dt + i_N(t_0)$$

$$= \left(\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}\right)\int_{t_0}^{t} v\,dt + i_1(t_0) + i_2(t_0)$$

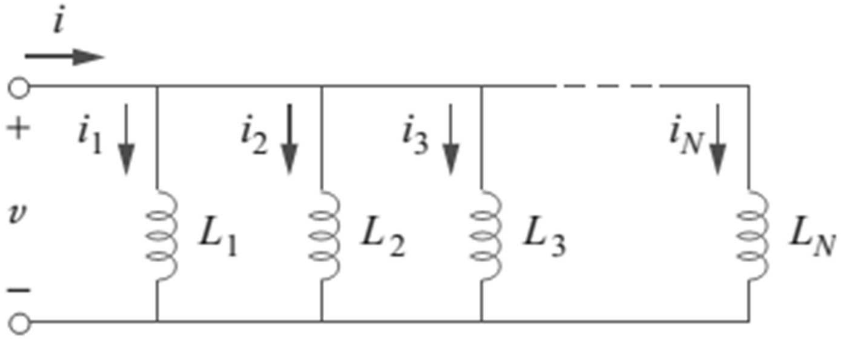
$$+ \cdots + i_N(t_0)$$

$$= \left(\sum_{k=1}^{N}\frac{1}{L_k}\right)\int_{t_0}^{t} v\,dt + \sum_{k=1}^{N} i_k(t_0) = \frac{1}{L_{eq}}\int_{t_0}^{t} v\,dt + i(t_0)$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$

SERIES AND PARALLEL INDUCTORS

The equivalent inductance of Parallel inductors is the reciprocal of the sum of the reciprocals of the individual inductances.

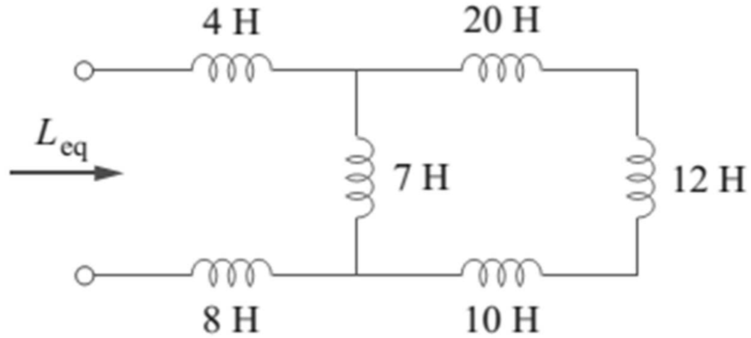


For two inductors connected in parallel;

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

PROBLEMS

Find the equivalent inductance?



(18 H)

CHARACTERISTICS OF BASIC ELEMENTS

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v\,dt + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

REFERENCES

Fundamentals of Electric Circuits (4th Edition)

Charles K. Alexander, Matthew N. O. Sadiku

Chapter 06 – Capacitors and Inductors (6.1 – 6.5)

Exercise Problems: 6.1 – 6.66

Do exercise problem yourself.